1 Exponents & Radicals

Here are the laws of exponents you should know:

Law	Example
$x^1 = x$	$3^1 = 3$
$x^0 = 1$	$3^0 = 1$
$x^m \cdot x^n = x^{m+n}$	$3^4 \cdot 3^5 = 3^9$
$\frac{x^m}{x^n} = x^{m-n}$	$\frac{3^7}{3^3} = 3^4$
$(x^m)^n = x^{mn}$	$(3^2)^4 = 3^8$
$(xy)^m = x^m y^m$	$(2 \cdot 3)^3 = 2^3 \cdot 3^3$
$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$
$x^{-m} = \frac{1}{x^m}$	$3^{-4} = \frac{1}{3^4}$

Many students don't know the difference between

$$(-3)^2 \text{ and } -3^2$$

Order of operations (PEMDAS) dictates that parentheses take precedence. So,

$$(-3)^2 = (-3) \cdot (-3) = 9$$

Without parentheses, exponents take precedence:

$$-3^2 = -3 \cdot 3 = -9$$

The negative is not applied until the exponent operation is carried through. Make sure you understand this so you don't make this common mistake. Sometimes, the result turns out to be the same, as in:

$$(-2)^3 \text{ and } -2^3$$

Make sure you see why they yield the same result.

EXERCISE 1: Evaluate WITHOUT a calculator. Answers for this chapter start on page 272.

1. $(-1)^4$
2. $(-1)^5$
3. $(-1)^{10}$
4. $(-1)^{15}$
5. $(-1)^8$
6. -1^8
7. $-(-1)^8$
8. $(-3)^3$
9. -3^3
10. $-(-3)^3$
11. $-(-6)^2$
12. $-(-4)^3$
13. $2^3 \times 3^2 \times (-1)^5$
14. $(-1)^4 \times 3^3 \times 2^2$
15. $(-2)^3 \times (-3)^4$
16. 3^0
17. 6^{-1}
18. 4^{-1}
19. 5^0
20. 3^2
21. 3^{-2}
22. 5^3
23. 5^{-3}
24. 7^2
25. 7^{-2}
26. 10^3
27. 10^{-3}

EXERCISE 2: Simplify so that your answer contains only positive exponents. Do NOT use a calculator. The first two have been done for you. Answers for this chapter start on page 272.

1. $3x^2 \cdot 2x^3 = 6x^5$
2. $2k^{-4} \cdot 4k^2 = \dfrac{8}{k^2}$
3. $5x^4 \cdot 3x^{-2}$
4. $7m^3 \cdot -3m^{-3}$
5. $(2x^2)^{-3}$
6. $-3a^2b^{-3} \cdot 3a^{-5}b^8$
7. $\dfrac{3n^7}{6n^3}$
8. $(a^2b^3)^2$
9. $\left(\dfrac{xy^4}{x^3y^2}\right)$
10. $-(-x)^3$
11. $(x^2y^{-1})^3$
12. $\dfrac{6u^4}{8u^2}$
13. $2uv^2 \cdot -4u^2v$
14. $\dfrac{x^2}{x^{-3}}$
15. $\dfrac{3x^4}{(x^{-2})^2}$
16. $\dfrac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$
17. $x^2 \cdot x^3 \cdot x^4$
18. $(x^2)^{-3} \cdot 2x^3$
19. $(2m)^2 \cdot (3m^3)^2$
20. $(a^{-1} \cdot a^{-2})^2$
21. $(b^{-2})^{-3} \cdot (b^3)^2$
22. $\dfrac{(m^2n)^3}{(mn^2)^2}$
23. $\dfrac{1}{x^{-2}}$
24. $\dfrac{mn}{m^2n^3}$
25. $\dfrac{k^{-2}}{k^{-3}}$
26. $\left(\dfrac{m^2}{n^3}\right)^3$
27. $\left(\dfrac{x^2y^3z^4}{x^{-3}y^{-4}z^{-5}}\right)$

EXAMPLE 1: If $3^{x+2} = y$, then what is the value of 3^x in terms of y ?

A) $y+9$ B) $y-9$ C) $\dfrac{y}{3}$ D) $\dfrac{y}{9}$

Let's avoid the trouble of finding what x is. Here we notice that the 2 in the exponent is the only difference between the given equation and what we want. So using our laws of exponents, let's extract the 2 out:

$$3^{x+2} = 3^x \cdot 3^2 = y$$
$$3^x = \frac{y}{9}$$

Answer $\boxed{(D)}$.

EXAMPLE 2: If $3^{a+1} = 3^{-a+7}$, what is the value of a ?

Here we see that the bases are the same. The exponents must therefore be equal.

$$a+1 = -a+7$$
$$2a = 6$$
$$a = \boxed{3}$$

EXAMPLE 3: If $2a - b = 4$, what is the value of $\dfrac{4^a}{2^b}$?

Realize that 4 is just 2^2.

$$\frac{4^a}{2^b} = \frac{(2^2)^a}{2^b} = \frac{2^{2a}}{2^b} = 2^{2a-b} = 2^4 = \boxed{16}$$

Square roots are just fractional exponents:

$$x^{\frac{1}{2}} = \sqrt{x}$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

But what about $x^{\frac{2}{3}}$? The 2 on top means to square x. The 3 on the bottom means to cube root it:

$$\sqrt[3]{x^2}$$

We can see this more clearly if we break it down:

$$x^{\frac{2}{3}} = (x^2)^{\frac{1}{3}} = \sqrt[3]{x^2}$$

The order in which we do the squaring and the cube-rooting doesn't matter.

$$x^{\frac{2}{3}} = (x^{\frac{1}{3}})^2 = (\sqrt[3]{x})^2$$

The end result just looks prettier with the cube root on the outside. That way, we don't need the parentheses.

EXAMPLE 4: Which of the following is equal to $\sqrt[4]{x^5}$?

A) x B) $x^5 - x^4$ C) $x^{\frac{5}{4}}$ D) $x^{\frac{4}{5}}$

The fourth root equates to a fractional exponent of $\dfrac{1}{4}$, so

$$\sqrt[4]{x^5} = x^{\frac{5}{4}}$$

Answer $\boxed{(C)}$.

The SAT will also test you on simplifying square roots (also called "surds"). To simplify a square root, factor the number inside the square root and take out any pairs:

$$\sqrt{48} = \sqrt{2\cdot 2\cdot 2\cdot 2\cdot 3} = \sqrt{\boxed{2\cdot 2}\cdot\boxed{2\cdot 2}\cdot 3} = 2\cdot 2\sqrt{3} = 4\sqrt{3}$$

In the example above, we take a 2 out for the first $\boxed{2\cdot 2}$. Then we take another 2 out for the second pair $\boxed{2\cdot 2}$. Finally, we multiply the two 2's outside the square root to get 4. Of course, a quicker route would have looked like this:

$$\sqrt{48} = \sqrt{\boxed{4\cdot 4}\cdot 3} = 4\sqrt{3}$$

Here's another example:

$$\sqrt{72} = \sqrt{\boxed{2\cdot 2}\cdot\boxed{3\cdot 3}\cdot 2} = 2\cdot 3\sqrt{2} = 6\sqrt{2}$$

To go backwards, take the number outside and put it back under the square root as a pair:

$$6\sqrt{2} = \sqrt{6\cdot 6\cdot 2} = \sqrt{72}$$

To simplify a cube root such as $\sqrt[3]{16}$, take out any triplets:

$$\sqrt[3]{16} = \sqrt[3]{\boxed{2\cdot 2\cdot 2}\cdot 2} = 2\sqrt[3]{2}$$

EXAMPLE 5: Which of the following is an equivalent form of $(x^2)^{\frac{3}{4}}$, where $x > 0$?

A) $\sqrt{x}$ B) $x\sqrt{x}$ C) $\sqrt[3]{x^2}$ D) $\sqrt[4]{x}$

Solution 1:

$$(x^2)^{\frac{3}{4}} = x^{2\cdot\frac{3}{4}} = x^{\frac{3}{2}} = \sqrt{x^3} = \sqrt{\boxed{x\cdot x}\cdot x} = x\sqrt{x}$$

Answer $\boxed{(B)}$.

Solution 2: Since $(x^2)^{\frac{3}{4}} = x^{2\cdot\frac{3}{4}} = x^{\frac{3}{2}}$, we can compare this exponent of $\dfrac{3}{2}$ to the exponent of x in each of the answer choices.

Choice A: $\sqrt{x} = x^{\frac{1}{2}}$

Choice B: $x\sqrt{x} = x^1\cdot x^{\frac{1}{2}} = x^{1+\frac{1}{2}} = x^{\frac{3}{2}}$

Choice C: $\sqrt[3]{x^2} = x^{\frac{2}{3}}$

Choice D: $\sqrt[4]{x} = x^{\frac{1}{4}}$

These results confirm that the answer is B.

EXERCISE 3: Simplify the radicals or solve for x. Do NOT use a calculator. Answers for this chapter start on page 272.

1. $\sqrt{12}$
2. $\sqrt{96}$
3. $\sqrt{45}$
4. $\sqrt{18}$
5. $2\sqrt{27}$
6. $3\sqrt{75}$
7. $\sqrt{32}$
8. $\sqrt{200}$
9. $\sqrt{8}$
10. $\sqrt{128}$
11. $5\sqrt{2} = \sqrt{x}$
12. $3\sqrt{x} = \sqrt{45}$
13. $2\sqrt{2} = \sqrt{4x}$
14. $4\sqrt{6} = 2\sqrt{3x}$
15. $3\sqrt{14} = \sqrt{6x}$
16. $4\sqrt{3x} = 2\sqrt{6}$
17. $3\sqrt{8} = x\sqrt{2}$
18. $x\sqrt{x} = \sqrt{216}$

CHAPTER EXERCISE: Answers for this chapter start on page 272.

A calculator should NOT be used on the following questions.

1

If $a^{-\frac{1}{2}} = 3$, what is the value of a ?

A) -9

B) $\frac{1}{9}$

C) $\frac{1}{3}$

D) 9

2

If $\frac{2^x}{2^y} = 2^3$, then x must equal

A) $y + 3$

B) $y - 3$

C) $3 - y$

D) $3y$

3

If $y^5 = 10$, what is the value of y^{20} ?

A) 40

B) 400

C) 1,000

D) 10,000

4

The expression $\sqrt[4]{x^2y^4}$, where $x > 0$ and $y > 0$, is equivalent to which of the following?

A) $\sqrt{xy}$

B) $y\sqrt{x}$

C) $\frac{1}{x^2}$

D) x^2y

5

If $\frac{\sqrt{x^3}}{\sqrt[4]{x}} = x^c$ for all positive values of x, what is the value of c ?

6

If $3^x = 10$, what is the value of 3^{x-3} ?

A) $\frac{10}{3}$

B) $\frac{10}{9}$

C) $\frac{10}{27}$

D) $\frac{27}{10}$

7

If a and b are positive even integers, which of the following is greatest?

A) $(-2a)^b$

B) $(-2a)^{2b}$

C) $(2a)^b$

D) $2a^{2b}$

8

If $x^2 = y^3$, for what value of z does $x^{3z} = y^9$?

A) -1

B) 0

C) 1

D) 2

9

If $\sqrt{x\sqrt{x}} = x^a$, then what is the value of a ?

A) $\frac{1}{2}$
B) $\frac{3}{4}$
C) 1
D) $\frac{4}{3}$

10

If $x^{ac} \cdot x^{bc} = x^{30}$, $x > 1$, and $a + b = 5$, what is the value of c ?

A) 3
B) 5
C) 6
D) 10

11

If $4^{2n+3} = 8^{n+5}$, what is the value of n ?

A) 6
B) 7
C) 8
D) 9

12

Which of the following is equivalent to $(-2)^{\frac{5}{3}}$?

A) $-2 \cdot \sqrt[3]{4}$
B) $2 \cdot \sqrt[3]{4}$
C) $-4 \cdot \sqrt[3]{2}$
D) $4 \cdot \sqrt[3]{2}$

13

If $2^{x+3} - 2^x = k(2^x)$, what is the value of k ?

A) 3
B) 5
C) 7
D) 8

A calculator is allowed on the following questions.

14

If $(5^3)^{4k} = (5^{\frac{1}{3}})^{24}$, what is the value of k ?

A) -6
B) $\frac{2}{3}$
C) $\frac{3}{4}$
D) 2

15

Which of the following is equivalent to $x^{\frac{2a}{b}}$ for all positive values of x, where a and b are positive integers?

A) $\sqrt[b]{ax^2}$
B) $\sqrt[b]{x^{2a}}$
C) $\sqrt[b]{x^{a+2}}$
D) $\sqrt[2a]{x^b}$

16

If $x^2y^3 = 10$ and $x^3y^2 = 8$, what is the value of x^5y^5 ?

A) 18
B) 20
C) 40
D) 80