# 1

# Exponents & Radicals

Given the expression $x^n$, $x$ is called the **base** and $n$ is called the **exponent** or **power**. Here are the laws of exponents you should know:

| Law | Example |
|---|---|
| $x^1 = x$ | $3^1 = 3$ |
| $x^0 = 1$ | $3^0 = 1$ |
| $x^m \cdot x^n = x^{m+n}$ | $3^4 \cdot 3^5 = 3^9$ |
| $\frac{x^m}{x^n} = x^{m-n}$ | $\frac{3^7}{3^3} = 3^4$ |
| $(x^m)^n = x^{mn}$ | $(3^2)^4 = 3^8$ |
| $(xy)^m = x^m y^m$ | $(2 \cdot 3)^3 = 2^3 \cdot 3^3$ |
| $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ | $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$ |
| $x^{-m} = \frac{1}{x^m}$ | $3^{-4} = \frac{1}{3^4}$ |

The difference between

$$(-3)^2 \text{ and } -3^2$$

comes down to order of operations (PEMDAS), which dictates that parentheses are prioritized first. So,

$$(-3)^2 = (-3) \cdot (-3) = 9$$

Without parentheses, exponents take priority:

$$-3^2 = -3 \cdot 3 = -9$$

Notice that the negative is not applied until the exponent operation is carried through.
Sometimes, the result turns out to be the same, as in:

$$(-2)^3 \text{ and } -2^3$$

Both yield $-8$.

**EXAMPLE 1:** Which of the following is equivalent to $-x^2(-x)^5$ ?

A) $-x^{10}$ B) $-x^7$ C) $x^7$ D) $x^{10}$

The $-x^2$ term cannot be simplified any further because the negative is not inside any parentheses. The $(-x)^5$ term has an odd exponent. An odd exponent yields an odd number of negative signs, which in turn yields a negative result. So, $(-x)^5$ simplifies to $-x^5$. The entire expression then boils down to

$$-x^2(-x)^5 = -x^2 \cdot -x^5 = x^{2+5} = x^7$$

Answer $\boxed{(C)}$.

**EXAMPLE 2:** Which expression is equivalent to $4^{x+1} \cdot 3^{2x}$, where $x > 0$ ?

A) $4(12)^x$ B) $4(24)^x$ C) $4(36)^x$ D) $8(12)^x$

We can use our laws of exponents to simplify the expression. First, the law $x^{m+n} = x^m \cdot x^n$ allows us to simplify $4^{x+1}$ to $4^x \cdot 4^1$. Then the law $x^{mn} = \left(x^m\right)^n$ allows us to turn $3^{2x}$ into $\left(3^2\right)^x = 9^x$. After applying these laws, we get

$$4^{x+1} \cdot 3^{2x} = 4^x \cdot 4^1 \cdot 9^x = 4 \cdot (4^x \cdot 9^x) = 4(36)^x$$

Answer $\boxed{(C)}$. In the last step, we used the law $x^m y^m = (xy)^m$ to get $4^x \cdot 9^x = 36^x$.

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Roots are the opposites of exponents. Whereas $5^2$ will give you 25, the square root of 25, denoted by $\sqrt{25}$, will give you back the 5. Similarly, taking the cube root of $6^3$ gives $\sqrt[3]{6^3} = 6$ and taking the fourth root of $b^4$ gives $\sqrt[4]{b^4} = b$. Notice how the operations cancel each other out.

Instead of using the radical sign, $\sqrt[n]{\phantom{x}}$, we can also denote the $n$th-root using the equivalent fractional exponent $\frac{1}{n}$. For example,

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

What about $x^{\frac{2}{3}}$ ? The 2 on top means to square $x$. The 3 on the bottom means to cube root it:

$$\sqrt[3]{x^2}$$

We can see this more clearly if we break it down using our laws of exponents:

$$x^{\frac{2}{3}} = \left(x^2\right)^{\frac{1}{3}} = \sqrt[3]{x^2}$$

The order in which we do the squaring and the cube-rooting doesn't matter. We could've broken it down this way as well:

$$x^{\frac{2}{3}} = (x^{\frac{1}{3}})^2 = (\sqrt[3]{x})^2$$

You'll see $\sqrt[3]{x^2}$ more often than $(\sqrt[3]{x})^2$ because the outside cube root avoids the need for parentheses.

**EXAMPLE 3:** Which of the following is equivalent to $\sqrt[4]{x^5}$ ?

A) $x$ B) $x^5 - x^4$ C) $x^{\frac{5}{4}}$ D) $x^{\frac{4}{5}}$

The fourth root equates to a fractional exponent of $\frac{1}{4}$, so

$$\sqrt[4]{x^5} = \left(x^5\right)^{\frac{1}{4}} = x^{\frac{5}{4}}$$

Answer $\boxed{\text{(C)}}$.

**EXAMPLE 4:** The expression $\frac{25^{2x}}{\left(\sqrt{5}\right)^x}$ is equivalent to $5^{kx}$, where $k$ is a constant. What is the value of $k$ ?

Because most of the laws of exponents involve terms with the same base, an often-used tactic is to convert each term to have the same base. Here, we're asked to simplify to $5^{kx}$, so we should convert everything to have the same base of 5. As is common in this type of question, we see that we have a perfect square, 25, which can be written as $5^2$. In the denominator, we have $\sqrt{5}$, which can be written as $5^{\frac{1}{2}}$.

$$\frac{25^{2x}}{\left(\sqrt{5}\right)^x} = \frac{(5^2)^{2x}}{(5^{\frac{1}{2}})^x} = \frac{5^{4x}}{5^{\frac{1}{2}x}} = 5^{4x-\frac{1}{2}x} = 5^{\frac{7}{2}x}$$

Therefore, $k = \boxed{\frac{7}{2}}$.

**EXAMPLE 5:** If the square of $a$ is equal to the cube of $b$, where $a \geq 0$ and $b \geq 0$, for what value of $x$ is $\sqrt{a^x}$ equal to $b$ ?

Essentially, we need to solve

$$\sqrt{a^x} = b$$

for $x$ given that $a^2 = b^3$. Let's first cube root both sides of $a^2 = b^3$ to isolate $b$:

$$\sqrt[3]{a^2} = \sqrt[3]{b^3}$$
$$a^{\frac{2}{3}} = b$$

Substituting $a^{\frac{2}{3}}$ for $b$ in the equation above, we get

$$\sqrt{a^x} = a^{\frac{2}{3}}$$
$$a^{\frac{1}{2}x} = a^{\frac{2}{3}}$$

Since both sides have the same base of $a$, we can just equate the exponents and solve for $x$:

$$\frac{1}{2}x = \frac{2}{3}$$
$$x = \boxed{\frac{4}{3}}$$

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The SAT will also test you on simplifying square roots (also called "surds"). To simplify a square root, factor the number inside the square root and take out any pairs:

$$\sqrt{48} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3} = \sqrt{\boxed{2 \cdot 2} \cdot \boxed{2 \cdot 2} \cdot 3} = 2 \cdot 2\sqrt{3} = 4\sqrt{3}$$

In the example above, we take a 2 out for the first $\boxed{2 \cdot 2}$. Then we take another 2 out for the second pair $\boxed{2 \cdot 2}$. Finally, we multiply the two 2's outside the square root to get 4 and leave the 3 inside. Of course, a quicker route would have been to break down 48 into bigger factor pairs:

$$\sqrt{48} = \sqrt{\boxed{4 \cdot 4} \cdot 3} = 4\sqrt{3}$$

Here's another example:

$$\sqrt{72} = \sqrt{\boxed{2 \cdot 2} \cdot \boxed{3 \cdot 3} \cdot 2} = 2 \cdot 3\sqrt{2} = 6\sqrt{2}$$

To go backwards, take the number outside and put it back under the square root as a pair:

$$6\sqrt{2} = \sqrt{6 \cdot 6 \cdot 2} = \sqrt{72}$$

To simplify a cube root such as $\sqrt[3]{16}$, take out any triplets:

$$\sqrt[3]{16} = \sqrt[3]{\boxed{2 \cdot 2 \cdot 2} \cdot 2} = 2\sqrt[3]{2}$$

**EXAMPLE 6:** Which of the following is equivalent to $\left(x^2\right)^{\frac{3}{4}}$, where $x > 0$ ?

A) $\sqrt{x}$ B) $x\sqrt{x}$ C) $\sqrt[3]{x^2}$ D) $\sqrt[4]{x}$

**Solution 1:**

$$\left(x^2\right)^{\frac{3}{4}} = x^{(2\cdot\frac{3}{4})} = x^{\frac{3}{2}} = \sqrt{x^3} = \sqrt{\boxed{x \cdot x} \cdot x} = x\sqrt{x}$$

Answer $\boxed{(B)}$.

**Solution 2:** Since $\left(x^2\right)^{\frac{3}{4}} = x^{(2\cdot\frac{3}{4})} = x^{\frac{3}{2}}$, we can compare this exponent of $\dfrac{3}{2}$ to the exponent of $x$ in each of the answer choices.

Choice A: $\sqrt{x} = x^{\frac{1}{2}}$

Choice B: $x\sqrt{x} = x^1 \cdot x^{\frac{1}{2}} = x^{(1+\frac{1}{2})} = x^{\frac{3}{2}}$

Choice C: $\sqrt[3]{x^2} = x^{\frac{2}{3}}$

Choice D: $\sqrt[4]{x} = x^{\frac{1}{4}}$

These results confirm that the answer is $\boxed{(B)}$.

**EXERCISE 1:** Evaluate WITHOUT a calculator. Answers for this exercise start on page 311.

1. $(-1)^{10}$
2. $(-1)^{15}$
3. $(-1)^{8}$
4. $-1^{8}$
5. $-(-1)^{8}$
6. $(-3)^{3}$
7. $-3^{3}$
8. $-(-6)^{2}$
9. $-(-4)^{3}$
10. $2^3 \times 3^2 \times (-1)^5$
11. $4^{-1}$
12. $5^{0}$
13. $3^{-2}$
14. $\left(\frac{1}{2}\right)^{-1}$
15. $\left(\frac{4}{3}\right)^{-2}$

**EXERCISE 2:** Simplify so that your answer contains only positive exponents. The first one has been done for you. Answers for this exercise start on page 312.

1. $2k^{-4} \cdot 4k^2 = \frac{2 \cdot 4k^2}{k^4} = \frac{8}{k^2}$
2. $3x^2 \cdot 2x^3$
3. $5x^4 \cdot 3x^{-2}$
4. $7m^3 \cdot -3m^{-3}$
5. $\left(2x^2\right)^{-3}$
6. $-3a^2b^{-3} \cdot 3a^{-5}b^8$
7. $\frac{3x^4}{\left(x^{-2}\right)^2}$
8. $\frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$
9. $(2m)^2 \cdot \left(3m^3\right)^2$
10. $\left(a^{-1} \cdot a^{-2}\right)^2$
11. $\left(b^{-2}\right)^{-3} \cdot \left(b^3\right)^2$
12. $\frac{\left(m^2n\right)^3}{\left(mn^2\right)^2}$
13. $\frac{mn}{m^2n^3}$
14. $\frac{k^{-2}}{k^{-3}}$
15. $\frac{x^2y^3z^4}{x^{-3}y^{-4}z^{-5}}$

**EXERCISE 3:** Simplify the radicals or solve for $x$. Answers for this exercise start on page 312.

1. $\sqrt{12}$
2. $\sqrt{96}$
3. $\sqrt{45}$
4. $\sqrt{18}$
5. $3\sqrt{75}$
6. $\sqrt{32}$
7. $5\sqrt{2} = \sqrt{x}$
8. $3\sqrt{x} = \sqrt{45}$
9. $2\sqrt{2} = \sqrt{4x}$
10. $4\sqrt{6} = 2\sqrt{3x}$
11. $3\sqrt{8} = x\sqrt{2}$
12. $x\sqrt{x} = \sqrt{216}$

**EXERCISE 4:** Answers for this exercise start on page 313.

**1**

Which expression is equivalent to $(x^2y)(x^4y^{-3})$, where $x$, $y$, and $z$ are positive numbers?

A) $x^6y^{-3}$

B) $x^6y^{-2}$

C) $x^8y^{-3}$

D) $x^8y^{-2}$

**2**

Which of the following is equivalent to $\sqrt{\frac{x}{64}}$ for all $x > 0$ ?

A) $\frac{x^2}{8}$

B) $\frac{x^2}{32}$

C) $\frac{x^{\frac{1}{2}}}{8}$

D) $\frac{x^{\frac{1}{2}}}{32}$

**3**

Which expression is equivalent to $\frac{x^{\frac{5}{6}}}{\sqrt[3]{x}}$, where $x \neq 0$ ?

A) $x^{\frac{1}{2}}$

B) $x^{\frac{5}{2}}$

C) $x^{\frac{2}{3}}$

D) $x^{\frac{5}{3}}$

**4**

Which expression is equivalent to $b^{\frac{9}{7}}$, where $b > 0$ ?

A) $\sqrt[15]{b^{105}}$

B) $\sqrt[15]{b^{135}}$

C) $\sqrt[105]{b^{135}}$

D) $\sqrt[135]{b^{105}}$

**5**

Which expression represents the product of $\left(b^6c^{-2}d^{-5}\right)$ and $\left(b^8c^{-3} + c^4d^5\right)$, where $b$, $c$, and $d$ are positive?

A) $b^{14}c^{-5} + c^2d^{-10}$

B) $b^{14}c^{-5} + c^2$

C) $b^{14}c^{-5}d^{-5} + b^6c^2$

D) $b^{14}c^{-5}d^{-5} + c^2$

**6**

If $x \neq 0$ and $y \neq 0$, which of the following is equivalent to $\frac{8x^2}{\sqrt{4x^6y^4}}$ ?

A) $2xy^{-2}$

B) $4x^{-2}y^2$

C) $4x^{-1}y^{-2}$

D) $4xy^2$

**7**

If $r$ and $s$ are positive, which of the following expressions is equivalent to $r^{\frac{6}{7}}s^{\frac{3}{7}}$ ?

A) $\dfrac{1}{\sqrt[6]{r^7s^{14}}}$

B) $\sqrt[6]{r^7s^{14}}$

C) $\dfrac{1}{\sqrt[7]{r^6s^3}}$

D) $\sqrt[7]{r^6s^3}$

**8**

$$\sqrt[b]{a} = \sqrt[d]{c}$$

The given equation relates the distinct positive numbers $a$, $b$, $c$, and $d$. Which equation correctly expresses $c$ in terms of $a$, $b$, and $d$ ?

A) $c = \dfrac{d}{a^{\frac{1}{b}}}$

B) $c = a^{d-b}$

C) $c = a^{\frac{b}{d}}$

D) $c = a^{\frac{d}{b}}$

**9**

Which expression is equivalent to $\sqrt{16a^{\frac{2}{3}}}$, where $a > 0$ ?

A) $4a^{\frac{1}{3}}$

B) $4a^{\frac{4}{3}}$

C) $8a^{\frac{1}{3}}$

D) $8a^{\frac{4}{3}}$

**10**

Which expression is equivalent to $\sqrt{rs}\,(\sqrt{r} + \sqrt{s})$, where $r \geq 0$ and $s \geq 0$ ?

A) $\sqrt{r^2s + rs^2}$

B) $r\sqrt{s} + s\sqrt{r}$

C) $rs\sqrt{r+s}$

D) $\sqrt{rs + r + s}$

**11**

The expression $\left(\sqrt[3]{x^2}\right)^n$, where $n$ is a constant, is equivalent to $x^8$. What is the value of $n$ ?

**12**

Which expression is equivalent to $2^{-2k} \cdot 3^k$ ?

A) $(3\sqrt{2})^k$

B) $\left(\dfrac{1}{36}\right)^k$

C) $\left(\dfrac{3}{4}\right)^k$

D) $\left(\dfrac{9}{4}\right)^k$

**13**

Which expression is equivalent to $a^{\frac{2}{9}}(a^{\frac{2}{3}})^{\frac{2}{3}}$, where $a$ is positive?

A) $\sqrt[3]{a^2}$

B) $\sqrt[3]{a^{16}}$

C) $\sqrt[9]{a^8}$

D) $\sqrt[9]{a^{14}}$

**14**

Which expression is equivalent to $\frac{4}{9}\left(\frac{2}{3}\right)^{a}$ ?

A) $\left(\frac{2}{3}\right)^{\frac{a}{2}}$

B) $\left(\frac{2}{3}\right)^{a-2}$

C) $\left(\frac{2}{3}\right)^{a+2}$

D) $\left(\frac{2}{3}\right)^{2a}$

**15**

$$\sqrt[4]{x^3} \cdot \sqrt[12]{x^5}$$

The given expression is equivalent to $\sqrt[m]{x^n}$, where $m$ and $n$ are positive constants each less than 10 and $x > 1$. What is the value of $\frac{m}{n}$ ?

**16**

Two numbers, $p$ and $q$, are each greater than zero, and the square of $p$ is equal to the cube root of $q$. For what value of $x$ is $p^{1-x}$ equal to the square root of $q$ ?

**EXERCISE 5:** Answers for this exercise start on page 314.

**1**

Which expression is equivalent to $\sqrt{100x^{36}}$ ?

A) $10x^6$
B) $50x^6$
C) $10x^{18}$
D) $50x^{18}$

**2**

If $(5^3)^{4k} = (5^{\frac{1}{3}})^{24}$, what is the value of $k$ ?

A) $-6$
B) $\frac{2}{3}$
C) $\frac{3}{4}$
D) 2

**3**

Which of the following is equivalent to $3y^{\frac{1}{2}}$ for all $y > 0$ ?

A) $\sqrt{3y}$
B) $\sqrt{9y}$
C) $\sqrt{\frac{3}{y}}$
D) $\sqrt{\frac{9}{y}}$

**4**

Which expression is equivalent to $4^{m+2}$ ?

A) $16^m$
B) $16 + 4^m$
C) $8(4^m)$
D) $16(4^m)$

**5**

Which expression is equivalent to $\sqrt[4]{x^2y^4}$, where $x > 0$ and $y > 0$ ?

A) $\sqrt{xy}$
B) $y\sqrt{x}$
C) $\frac{1}{x^2}$
D) $x^2y$

**6**

Which expression is equivalent to $p^{\frac{1}{3}} \cdot (\sqrt[3]{p})^2$, where $p$ is greater than 0 ?

A) $p^{\frac{2}{3}}$
B) $p^{\frac{2}{9}}$
C) $p^{\frac{7}{3}}$
D) $p$

**7**

Which expression is equivalent to $x^{\frac{2a}{b}}$ for all positive values of $x$, where $a$ and $b$ are positive integers?

A) $\sqrt[b]{ax^2}$
B) $\sqrt[b]{x^{2a}}$
C) $\sqrt[b]{x^{a+2}}$
D) $\sqrt[2a]{x^b}$

**8**

If $\frac{9^7}{\sqrt[4]{9^{10}}} = 9^{6t}$, what is the value of $t$ ?

**9**

Which expression is equivalent to $a^{\frac{3}{4}}b^{\frac{1}{2}}$, where $a \geq 0$ and $b \geq 0$ ?

A) $\sqrt[4]{a^3b}$

B) $\sqrt[4]{a^3b^2}$

C) $\sqrt{a^3b}$

D) $\sqrt{a^4b^2}$

**10**

Which expression is equivalent to $h^{\frac{5}{12}}(h^{-\frac{1}{4}})^{\frac{7}{3}}$, where $h > 0$ ?

A) $\dfrac{1}{h^6}$

B) $\sqrt{h^5}$

C) $\dfrac{1}{\sqrt[6]{h}}$

D) $\dfrac{1}{\sqrt[5]{h^2}}$

**11**

$$\sqrt[8]{41k}\left(\sqrt[9]{41k}\right)^{12}$$

For what value of $x$ is the given expression equivalent to $(41k)^{25x}$, where $k > 1$ ?

**12**

The expression $4^{18x}$ is equivalent to $k^{3x}$, where $k$ is a constant. What is the value of $k$ ?

**13**

Which expression is equivalent to $9^{\frac{1}{n}}(4^{\frac{1}{2n}})$, where $n$ is a positive integer?

A) $24^{\frac{1}{n}}$

B) $12^{\frac{1}{n}}$

C) $\sqrt[n]{18}$

D) $\sqrt[n]{6}$

**14**

$$\frac{10\sqrt[6]{9^3x^{48}}}{\sqrt[4]{6^4x}}$$

The given expression is equivalent to $ax^b$, where $a > 0$, $b > 0$, and $x > 1$. What is the value of $ab$ ?

**15**

If $m$ and $n$ are both positive numbers, and $4m$ is equal to the cube root of the square of $n$, for what value of $x$ is $m^x$ equal to $n$ when $m = 2$ ?

**16**

If $2^{x+3} - 2^x = k(2^x)$, what is the value of $k$ ?

A) 3

B) 5

C) 7

D) 8