1

Exponents & Radicals

Here are the laws of exponents you should know:

Law	**Example**
$x^1 = x$	$3^1 = 3$
$x^0 = 1$	$3^0 = 1$
$x^m \cdot x^n = x^{m+n}$	$3^4 \cdot 3^5 = 3^9$
$\dfrac{x^m}{x^n} = x^{m-n}$	$\dfrac{3^7}{3^3} = 3^4$
$(x^m)^n = x^{mn}$	$(3^2)^4 = 3^8$
$(xy)^m = x^m y^m$	$(2 \cdot 3)^3 = 2^3 \cdot 3^3$
$\left(\dfrac{x}{y}\right)^m = \dfrac{x^m}{y^m}$	$\left(\dfrac{2}{3}\right)^3 = \dfrac{2^3}{3^3}$
$x^{-m} = \dfrac{1}{x^m}$	$3^{-4} = \dfrac{1}{3^4}$

Many students don't know the difference between

$$(-3)^2 \text{ and } -3^2$$

Order of operations (PEMDAS) dictates that parentheses take precedence. So,

$$(-3)^2 = (-3) \cdot (-3) = 9$$

Without parentheses, exponents take precedence:

$$-3^2 = -3 \cdot 3 = -9$$

The negative is not applied until the exponent operation is carried through. Make sure you understand this so you don't make this common mistake. Sometimes, the result turns out to be the same, as in:

$$(-2)^3 \text{ and } -2^3$$

Make sure you see why they yield the same result.

EXERCISE 1: Evaluate WITHOUT a calculator. Answers for this chapter start on page 251.

1. $(-1)^4$
2. $(-1)^5$
3. $(-1)^{10}$
4. $(-1)^{15}$
5. $(-1)^8$
6. -1^8
7. $-(-1)^8$
8. $(-3)^3$
9. -3^3
10. $-(-3)^3$
11. $-(-6)^2$
12. $-(-4)^3$
13. $2^3 \times 3^2 \times (-1)^5$
14. $(-1)^4 \times 3^3 \times 2^2$
15. $(-2)^3 \times (-3)^4$
16. 3^0
17. 6^{-1}
18. 4^{-1}
19. 5^0
20. 3^2
21. 3^{-2}
22. 5^3
23. 5^{-3}
24. 7^2
25. 7^{-2}
26. 10^3
27. 10^{-3}

EXERCISE 2: Simplify so that your answer contains only positive exponents. Do NOT use a calculator. The first two have been done for you. Answers for this chapter start on page 251.

1. $3x^2 \cdot 2x^3 = 6x^5$
2. $2k^{-4} \cdot 4k^2 = \dfrac{8}{k^2}$
3. $5x^4 \cdot 3x^{-2}$
4. $7m^3 \cdot -3m^{-3}$
5. $(2x^2)^{-3}$
6. $-3a^2b^{-3} \cdot 3a^{-5}b^8$
7. $\dfrac{3n^7}{6n^3}$
8. $(a^2b^3)^2$
9. $\left(\dfrac{xy^4}{x^3y^2}\right)$
10. $-(-x)^3$
11. $(x^2y^{-1})^3$
12. $\dfrac{6u^4}{8u^2}$
13. $2uv^2 \cdot -4u^2v$
14. $\dfrac{x^2}{x^{-3}}$
15. $\dfrac{3x^4}{(x^{-2})^2}$
16. $\dfrac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$
17. $x^2 \cdot x^3 \cdot x^4$
18. $(x^2)^{-3} \cdot 2x^3$
19. $(2m)^2 \cdot (3m^3)^2$
20. $(a^{-1} \cdot a^{-2})^2$
21. $(b^{-2})^{-3} \cdot (b^3)^2$
22. $\dfrac{(m^2n)^3}{(mn^2)^2}$
23. $\dfrac{1}{x^{-2}}$
24. $\dfrac{mn}{m^2n^3}$
25. $\dfrac{k^{-2}}{k^{-3}}$
26. $\left(\dfrac{m^2}{n^3}\right)^3$
27. $\left(\dfrac{x^2y^3z^4}{x^{-3}y^{-4}z^{-5}}\right)$

EXAMPLE 1: If $3^{x+2} = y$, then what is the value of 3^x in terms of y?

A) $y+9$ B) $y-9$ C) $\dfrac{y}{3}$ D) $\dfrac{y}{9}$

Let's avoid the trouble of finding what x is. Here we notice that the 2 in the exponent is the only difference between the given equation and what we want. So using our laws of exponents, let's extract the 2 out:

$$3^{x+2} = 3^x \cdot 3^2 = y$$

$$3^x = \frac{y}{9}$$

The answer is $\boxed{(D)}$.

EXAMPLE 2: If $3^{a+1} = 3^{-a+7}$, what is the value of a?

Here we see that the bases are the same. The exponents must therefore be equal.

$$a + 1 = -a + 7$$

$$2a = 6$$

$$a = \boxed{3}$$

EXAMPLE 3: If $2a - b = 4$, what is the value of $\dfrac{4^a}{2^b}$?

Realize that 4 is just 2^2.

$$\frac{4^a}{2^b} = \frac{(2^2)^a}{2^b} = \frac{2^{2a}}{2^b} = 2^{2a-b} = 2^4 = \boxed{16}$$

Square roots are just fractional exponents:

$$x^{\frac{1}{2}} = \sqrt{x}$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

But what about $x^{\frac{2}{3}}$? The 2 on top means to square x. The 3 on the bottom means to cube root it:

$$\sqrt[3]{x^2}$$

We can see this more clearly if we break it down:

$$x^{\frac{2}{3}} = (x^2)^{\frac{1}{3}} = \sqrt[3]{x^2}$$

The order in which we do the squaring and the cube-rooting doesn't matter.

$$x^{\frac{2}{3}} = (x^{\frac{1}{3}})^2 = (\sqrt[3]{x})^2$$

The end result just looks prettier with the cube root on the outside. That way, we don't need the parentheses.

EXAMPLE 4: Which of the following is equal to $\sqrt[4]{x^5}$?

A) x B) $x^5 - x^4$ C) $x^{\frac{5}{4}}$ D) $x^{\frac{4}{5}}$

The fourth root equates to a fractional exponent of $\dfrac{1}{4}$, so

$$\sqrt[4]{x^5} = x^{\frac{5}{4}}$$

Answer $\boxed{(C)}$.

The SAT will also test you on simplifying square roots (also called “surds”). To simplify a square root, factor the number inside the square root and take out any pairs:

$$\sqrt{48} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3} = \sqrt{\boxed{2 \cdot 2} \cdot \boxed{2 \cdot 2} \cdot 3} = 2 \cdot 2\sqrt{3} = 4\sqrt{3}$$

In the example above, we take a 2 out for the first $\boxed{2 \cdot 2}$. Then we take another 2 out for the second pair $\boxed{2 \cdot 2}$. Finally, we multiply the two 2’s outside the square root to get 4. Of course, a quicker route would have looked like this:

$$\sqrt{48} = \sqrt{\boxed{4 \cdot 4} \cdot 3} = 4\sqrt{3}$$

Here’s one more example:

$$\sqrt{72} = \sqrt{\boxed{2 \cdot 2} \cdot \boxed{3 \cdot 3} \cdot 2} = 2 \cdot 3\sqrt{2} = 6\sqrt{2}$$

To go backwards, take the number outside and put it back under the square root as a pair:

$$6\sqrt{2} = \sqrt{6 \cdot 6 \cdot 2} = \sqrt{72}$$

EXAMPLE 5: If $4\sqrt{3} = \sqrt{3x}$, what is the value of x?

A) 4 B) 12 C) 16 D) 48

Solution 1: Moving the 4 back inside, we get

$$4\sqrt{3} = \sqrt{3 \cdot 4 \cdot 4} = \sqrt{48}$$

Now equating the stuff inside the square roots,

$$\sqrt{48} = \sqrt{3x}$$

$$48 = 3x$$

$$16 = x$$

Answer $\boxed{(C)}$.

Solution 2: Square both sides:

$$(4\sqrt{3})^2 = (\sqrt{3x})^2$$
$$16 \cdot 3 = 3x$$
$$16 = x$$

EXERCISE 3: Simplify the radicals or solve for x. Do NOT use a calculator. Answers for this chapter start on page 251.

1. $\sqrt{12}$
2. $\sqrt{96}$
3. $\sqrt{45}$
4. $\sqrt{18}$
5. $2\sqrt{27}$
6. $3\sqrt{75}$
7. $\sqrt{32}$
8. $\sqrt{200}$
9. $\sqrt{8}$
10. $\sqrt{128}$
11. $5\sqrt{2} = \sqrt{x}$
12. $3\sqrt{x} = \sqrt{45}$
13. $2\sqrt{2} = \sqrt{4x}$
14. $4\sqrt{6} = 2\sqrt{3x}$
15. $3\sqrt{14} = \sqrt{6x}$
16. $4\sqrt{3x} = 2\sqrt{6}$
17. $3\sqrt{8} = x\sqrt{2}$
18. $x\sqrt{x} = \sqrt{216}$

CHAPTER EXERCISE: Answers for this chapter start on page 251.

A calculator should NOT be used on the following questions.

1

If $a^{-\frac{1}{2}} = 3$, what is the value of a?

A) -9

B) $\frac{1}{9}$

C) $\frac{1}{3}$

D) 9

2

$$\text{Let } n = 1^2 + 1^4 + 1^6 + 1^8 + \ldots + 1^{50}$$

What is the value of n?

A) 10

B) 20

C) 25

D) 30

3

If $4^{2n+3} = 8^{n+5}$, what is the value of n?

A) 6

B) 7

C) 8

D) 9

4

If $\frac{2^x}{2^y} = 2^3$, then x must equal

A) $y + 3$

B) $y - 3$

C) $3 - y$

D) $3y$

5

If $3^x = 10$, what is the value of 3^{x-3}?

A) $\frac{10}{3}$

B) $\frac{10}{9}$

C) $\frac{10}{27}$

D) $\frac{27}{10}$

6

If $x^2y^3 = 10$ and $x^3y^2 = 8$, what is the value of x^5y^5?

A) 18

B) 20

C) 40

D) 80

7

If a and b are positive even integers, which of the following is greatest?

A) $(-2a)^b$

B) $(-2a)^{2b}$

C) $(2a)^b$

D) $2a^{2b}$

8

Which of the following is equivalent to $x^{\frac{2a}{b}}$, for all values of x?

A) $\sqrt[b]{ax^2}$

B) $\sqrt[b]{x^{2a}}$

C) $\sqrt[b]{x^{a+2}}$

D) $\sqrt[2a]{x^b}$

9

If $x^2 = y^3$, for what value of z does $x^{3z} = y^9$?

A) -1
B) 0
C) 1
D) 2

10

If $2^{x+3} - 2^x = k(2^x)$, what is the value of k?

A) 3
B) 5
C) 7
D) 8

11

If $\sqrt{x\sqrt{x}} = x^a$, then what is the value of a?

A) $\frac{1}{2}$
B) $\frac{3}{4}$
C) 1
D) $\frac{4}{3}$

12

$$2\sqrt{x+2} = 3\sqrt{2}$$

If $x > 0$ in the equation above, what is the value of x?

A) 2.5
B) 3
C) 3.5
D) 4

13

If $x^{ac} \cdot x^{bc} = x^{30}$, $x > 1$, and $a + b = 5$, what is the value of c?

A) 3
B) 5
C) 6
D) 10

A calculator is allowed on the following questions.

14

If $n^3 = x$ and $n^4 = 20x$, where $n > 0$, what is the value of x?

15

If $x^8y^7 = 333$ and $x^7y^6 = 3$, what is the value of xy?